Abstract
Due to changes in fluid properties, porosity log responses change with pressure, especially for gas/water systems. Velocity data (compressional and shear) can be described using the Krief rock physics model. This model involves relationships among compressional and shear velocities, rock bulk density, elastic moduli of the matrix, shale and fluid components, shear modulus of the solids, and Biot coefficients. Density response is governed by changing gas density as a function of pressure. Neutron response is controlled by hydrogen indices of the fluids and the excavation effect.

A complete porosity log suite model is presented whereby pseudo porosity logs are calculated as pressure and gas saturation dependent functions.

Matrix and shale properties are included. Additionally mechanical properties with changing pressure and saturation are available.

From compressional travel time and density log responses, changes in synthetic seismograms as reservoir pressure is reduced can be calculated. This application has significant implications in the interpretation of 4-D seismic surveys over reservoirs undergoing pressure depletion.

Estimates of changing mechanical properties as functions of saturation and pressure have a number of engineering applications, including stimulation design and sand control.

Examples from a variety of clastic and carbonate reservoirs are presented, including intermediate depth hard rocks, deep offshore soft rocks, and shallow onshore soft rocks.

Introduction
Krief (1989) and Gassmann (1951)/Biot (1956) have independently developed rock physics models to account for velocity changes as gas saturation varies. A comparison of the two models, expressed in petrophysical terms, has been published by Holmes, et al (2004). Inherent in both models are effects of pressure on velocity responses, as a function of changing bulk moduli of the contained fluids (particularly gas). Pressure effects on density and neutron log responses in gas/water systems are a consequence of changing gas density and hydrogen indices.

In this paper, the Krief and Gassmann models are expressed in petrophysical terms and extended to include neutron log responses. Pseudo logs are calculated for ranges of gas/water combinations and pressure:
- Acoustic compressional
- Acoustic shear
- Density
- Neutron

These in turn can be used to estimate mechanical properties.

The porosity modeling assumes an isotropic medium, and that porosity does not change with pressure variations.

Basic Equations
Listed below are equations used to calculate pseudo porosity log responses from a petrophysical model of matrix/porosity/shale (a list of the nomenclature is available at the end of the paper):
Gassmann/Biot
To calculate DTC and DTS:

\[
\frac{K_{sat}}{K_o - K_{sat}} = \frac{K_{dry}}{K_o - K_{dry}} + \frac{K_{fl}}{\phi (K_o - K_{fl})} \quad \text{(1)}
\]

Assume \( \mu_{sat} = \mu_{dry} \)  \( \text{(2)} \)

\[
K_{dry} = K_o \times (1 - \phi)^{\frac{1}{1-\phi}} \quad \text{(3)}
\]

Krief
To calculate DTC and DTS:

\[
V_p^2 = \frac{k + \frac{4}{3} \mu}{\rho_B} \quad \text{(4)}
\]

\[
K = K_{ma} + (1 - \beta_B) + \beta^2 M_B \quad \text{(5)}
\]

\[
\mu = \mu_{ma} \times (1 - \beta_B) = V_s^2 \times \rho_B \quad \text{(6)}
\]

\[
\sqrt{M_B} = \frac{\beta_B - \phi}{K_{ma}} + \frac{\phi}{K_f} \quad \text{(7)}
\]

\[
1 - \beta_B = (1 - \phi)^{\frac{1}{1-\phi}} \quad \text{(8)}
\]

\[
M = V_p^2 \times \rho_B \quad \text{(9)}
\]

Density Log
For the clean (non-shaley) formation part of the system, termed here \( \phi_e \):

\[
\phi_e = \frac{\rho_{ma} - \rho_B}{\rho_{ma} - \rho_{fl}} \quad \text{(10)}
\]

\[
\rho_{fl} = \rho_w \times S_w + \rho_g \times (1 - S_w) \quad \text{(11)}
\]

Our modeling incorporates shale responses of the density log to properly account for shale responses. We have found good results for shale volumes up to 70%. Above this value, we empirically assign pseudo log response as equal to actual log response.

Neutron Log
For the clean formation part of the system:

\[
\phi_n = \left[H_w \times S_{so} + H_h \times (1 - S_{so})\right] \times \phi_e \quad \text{(12)}
\]

Our modeling incorporates shale responses of the neutron log to properly account for shale. We have found good results for shale volume up to 70%. Above this value, we empirically assign the pseudo log response as equal to actual log response.

Modeling Procedures

Initial Porosity-Matrix-Shale Model
An initial porosity-matrix-shale model is created, preferably using the following procedures:

- Porosity from a density/neutron cross plot. This porosity source involves no assumptions of matrix or fluid properties, since the cross plot porosity is relatively insensitive to either of these parameters.
- Shale volume from Gamma Ray or SP log.

A check on the validity of the calculations is compared with core-measured porosities (if available).

Select Reservoir Parameters
Matrix, shale and fluid properties and initial reservoir pressure input are required as follows:

- Compressional and shear travel times for matrix and shale. Usually this is specific by petrophysical zone.
- Bulk moduli for the contained fluids – water and gas. A generalized algorithm was developed for water bulk modulus, which is pressure, salinity and dissolved gas dependent. For gas, we have written a pressure-dependent algorithm, based on average gas properties. Both algorithms were derived with data from Craft and Hawkins (1959).
- Matrix and shale properties for the density and neutron logs – again petrophysical zone specific.
- Reservoir pressure appropriate to the time that the interval was logged.

Run the Fluid Substitution Models
The Krief and Gassmann/Biot models can then be run using assumptions as to the gas/water mix; both give closely comparable results. We have formulated calculations at consistent values as follows:

- \( S_g = 0\% = \text{Wet Formation, in 10\% increments of } S_g \text{ to } S_g = 80\% \)

Compare Pseudo Logs with Actual Logs
A detailed comparison of pseudo logs at \( S_g = 0\% \) and \( S_g = 80\% \) is then made with actual logs. If matrix and shale picks are correct, then the actual curve response must fall between the pseudo curves of \( S_g = 0\% \) and \( S_g = 80\% \) - assuming a reasonable minimum \( S_w \) of 20%. Figure 1 is a schematic of the approach.
Figure 1: Schematic demonstrating the determination of gas saturations from the pseudo logs. The pseudo logs provide a range of possible values, and the actual log response within this range is the gas value.

If the actual curve falls outside the pseudo curve limits the most likely explanation is an incorrect choice for one or more of the matrix or shale input parameters. Appropriate changes in zone input are then required, followed by rerunning the program. Occasional minor misfits are to be expected, as a consequence of local rock property changes and/or bad hole effects on log responses.

By comparing actual log response with the pseudo logs, gas saturations as “seen” by each of the porosity logs are available as continuous curves. Comparisons with standard resistivity modeling gives insight into other petrophysical attributes – degree of mud filtrate invasion, formation permeability, formation water salinity in intervals when this parameter varies and/or if it is not well defined.

Once matrix and shale parameters have been verified for a reservoir sequence, the pseudo curves are a reliable measure of acoustic properties, even when no acoustic measurements have been made. Presuming a density and neutron combination has been run then the pseudo acoustic compressional and shear curves can be used with confidence. Consequently, a full spectrum of mechanical properties is routinely available, without the need for empirical assumptions as to shear/compressional relations:

- Young’s Modulus
- Bulk Modulus
- Shear Modulus
- Poisson’s Ratio

All of these calculations require the following input:
- Compressional Velocity
- Shear Velocity
- Bulk Density

Review Pressure Effects on Porosity Logs
Once the initial model has been correctly established, pressure effects on porosity log response can be examined by changing the gas and water bulk moduli, gas density, and gas hydrogen index. This modeling will show the progressive pseudo log response changes as pressure is reduced. Changes of mechanical properties can also be monitored as a function of pressure.

Create Synthetic Seismograms
Output from the same calculation procedures can be used to quantify changes in synthetic seismogram response as a function of pressure.

Examples
Examples from four reservoirs are presented:
- Shallow High-Porosity Cretaceous Sandstone, SW Wyoming
- Offshore Gulf of Mexico
- Tight Gas Sandstone, SW Wyoming
- Carbonate from the Texas Panhandle

All calculations were made using the Krief model.

Porosity Log Changes with Pressure

Shallow High-Porosity Cretaceous Sandstone, SW Wyoming (Figures 2a-c)
DT compressional increases significantly as pressure is reduced; there is a negligible change in DT shear. Extreme reduction in neutron porosity and a decrease in bulk density are observed for gas-saturated pseudo logs. Wet curves for both neutron and density logs do not change with pressure.

Offshore Gulf of Mexico (Figures 3a-c)
Results are generally similar to the Shallow High-Porosity Cretaceous Sandstone responses, as to be expected.

Tight Gas Sandstone, SW Wyoming (Figures 4a-c)
Insignificant changes in DT compressional as pressure is reduced – due to the very small volumes of gas in these low-porosity sandstones. Significant changes in both neutron and density logs, but less than in the high-porosity reservoirs described previously (since pore volume is much smaller).

Carbonate from the Texas Panhandle (Figures 5a-c)
This example has similar responses to the tight gas sandstones from SW Wyoming, since the porosity magnitude is about the same. Matrix and shale input are quite different, because this is a Paleozoic carbonate sequence (mostly dolomite).
Mechanical Properties Changes with Pressure

**Shallow High-Porosity Sandstone, SW Wyoming (Figures 6a-b)**
Subtle changes in mechanical properties occur as pressure is reduced. Bulk modulus is reduced – particularly for wet formation. Young’s modulus and Shear modulus show very little change. Poisson’s Ratio is reduced – both gas-bearing and wet rocks.

**Offshore Gulf of Mexico (Figures 7a-b)**
As in the previous example, there are only subtle changes in rock properties as pressure is reduced, with a similar result in the Bulk modulus, Young’s modulus, Shear modulus and Poisson’s ratio.

Synthetic Seismogram Changes with Pressure

**Offshore Gulf of Mexico (Figures 8a-c)**
As pressure is reduced, two-way times increase, but relative seismic signature is almost invariant.

**Tight Gas Sandstone, SW Wyoming (Figures 9a-c)**
As pressure is reduced there are significant changes in synthetic seismogram responses. This suggests subtle controls on acoustic impedance as pressure changes – mostly a consequence of density changes.
Figure 2a: Shallow High-Porosity Sandstone, SW Wyoming at 10000 psi. For the interval 125-140ft, observe the pseudo porosity log response changes as pressure is reduced (Figures 2b-c). Also note that the wet pseudo neutron and density logs do not change with pressure, whereas the wet pseudo acoustic compressional does change – due to changing water compressibility. The acoustic shear shows insignificant changes with pressure.
Figure 2b: Shallow High-Porosity Sandstone, SW Wyoming at 5000 psi.
Figure 2c: Shallow High-Porosity Sandstone, SW Wyoming at 100 psi. Actual reservoir pressure is about 100 psi.
Figure 3a: Offshore Gulf of Mexico at 10000 psi. Actual reservoir pressure is about 10000 psi. Note the large changes in pseudo density and neutron curves (gas bearing) as pressure is reduced. Pseudo acoustic compressional, both gas and wet, change with pressure. Acoustic shear is not pressure dependant.
Figure 3b: Offshore Gulf of Mexico at 6000 psi.
**Figure 3c: Offshore Gulf of Mexico at 2000 psi.**
Figure 4a: Tight Gas Sandstone, SW Wyoming at 10000 psi. There are insignificant changes in pseudo acoustic curves as pressure is reduced, and no separation between wet and gas – a consequence of low porosity. Also note significant changes in pseudo neutron and density logs (gas-bearing) as pressure is reduced.
Figure 4b: Tight Gas Sandstone, SW Wyoming at 6000 psi. Actual reservoir pressure is about 6000 psi.
Figure 4c: Tight Gas Sandstone, SW Wyoming at 2000 psi.
Figure 5a: Carbonate from the Texas Panhandle at 10000 psi. As pressure is reduced, pseudo neutron and density logs (gas-bearing) show significant changes. Pseudo acoustic compressional data shows no separation due to gas (low porosity) and insignificant changes as pressure is reduced.
Figure 5b: Carbonate from the Texas Panhandle at 5000 psi.
Figure 5c: Carbonate from the Texas Panhandle at 1000 psi. Actual reservoir pressure is less than 100 psi.
Figure 6a: Shallow High-Porosity Sandstone, SW Wyoming at 10000 psi. Note the changes in Bulk Modulus and Poisson’s Ratio as pressure is reduced.
Figure 6b: Shallow High-Porosity Sandstone, SW Wyoming at 100 psi.
Figure 7a: Offshore Gulf of Mexico at 10000 psi. Note the changes in Bulk modulus and Poisson's ratio as pressure is reduced.
Figure 7b: Offshore Gulf of Mexico at 2000 psi.
Figure 8a: Offshore Gulf of Mexico at 10000 psi. Note the increase in two-way time as pressure is decreased.

Figure 8b: Offshore Gulf of Mexico at 6000 psi.
PSI: 2000

Figure 8c: Offshore Gulf of Mexico at 2000 psi.

PSI: 10000

Figure 9a: Tight Gas Sandstone, SW Wyoming at 10000 psi. Note the increase in two-way time and changes in seismic response as pressure is decreased.
Figure 9b: Tight Gas Sandstone, SW Wyoming at 6000 psi.

Figure 9c: Tight Gas Sandstone, SW Wyoming at 2000 psi.
Applications

Geophysical
The two synthetic seismogram examples suggest that changing seismic signatures, as evidenced by synthetic seismograms, are sufficiently emphatic that they should be observed in 4-D seismic surveys. By suitable calibration to well data, it might be possible in gas reservoirs to map the degree of reservoir pressure depletion away from areas of well control.

Engineering
Changing mechanical properties as pressure declines, and as a function of gas saturation, can be mapped. This should mean that the changing environment of wellbore stability and, in unconsolidated sands, the severity of sand production, can be modeled through the history of pressure depletion. Additionally, changing mechanical properties affect stimulation design.

Conclusions
Techniques are presented to create pseudo porosity logs as a function of changing fluid saturation (gas vs. water) and reservoir pressure:
- Acoustic Compressional
- Acoustic Shear
- Density
- Neutron

The calculations are based on the well-known rock physics geophysical models of Krief and Gassmann/Biot adapted into the petrophysical environment.

Other applications are the ability to calculate pseudo acoustic data (both compressional and shear), once the stratigraphic sequence has been calibrated, even where no acoustic measurements have been made. As a consequence, reliable mechanical properties can be calculated in any assumed gas/water mixture, and at any pressure. Wellbore stability and sand control issues can be modeled, and data used for stimulation design.

A potential application to geophysics is creating synthetic seismograms as a function of pressure change, and calibration of 4-D seismic as reservoir pressure is depleted.

Nomenclature

DTC  Compressional travel time
DTS  Shear travel time
\( H_h \)  Hydrogen index of hydrocarbons – dependent on the type of hydrocarbon, and heavily pressure dependent
\( H_W \)  Hydrogen index of water – varies with water salinity, but not pressure
\( K \)  Bulk modulus
\( K_{dry} \)  Bulk modulus of dry rock
\( K_f \)  Bulk modulus of fluid
\( K_{ma} \)  Bulk modulus of matrix (Krief)
\( K_o \)  Bulk modulus of matrix (Gassmann)
\( K_{sat} \)  Bulk modulus of saturated rock
\( M \)  Elastic modulus
\( S_W \)  Water saturation
\( S_{so} \)  Water saturation of the zone as “seen” by the neutron log – often assumed to be flushed zone saturation
\( \beta_B \)  Biot compressibility constant
\( \phi \)  Porosity
\( \phi_e \)  Effective porosity
\( \phi_n \)  Porosity from the neutron log
\( \mu \)  Shear modulus
\( \mu_{sat} \)  Shear modulus of saturated rock
\( \rho_B \)  Bulk density
\( \rho_fl \)  Fluid density
\( \rho_g \)  Gas density – heavily pressure dependent
\( \rho_{ma} \)  Matrix density
\( \rho_W \)  Water density – does not vary greatly with pressure

References